What's metastability (in neural networks)?

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NEUROMAT

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Outline

The concept of metastability

Metastability in a system of spiking neurons

What is Metastability in general?

Metastability is a very general phenomenon that occurs in a wide variety of fields, and probably initially came from physics. Here the definition from the metastability entry on Wikipedia (fr):

"Metastability is the property of a state, seemingly stable, but such that a tiny perturbation can push it toward an even more stable state."

The mathematician and physicist Bernard Derrida also used the expression "Dynamical phase transition".

What is Metastability in general?

We can find a wide variety of examples from different field in which metastability arises:

1. Biology: Carbon compounds in human body are in a metastable state, from which they can escape thanks to the catalytic reaction triggered by enzyme.
2. Physic: Supercooling water, avalanche, nuclear physics...
3. Digital Electronics: Metastable bits.
4. Mathematics: Catastrophe Theory in differential topology, Dynamical systems, Stochastic processes...
5. Existentialist Ontology: ?

What is Metastability in the brain?

"When we take a general view of the wonderful stream of our consciousness, what strikes us first is the different pace of its parts. Like a bird's life, it seems to be made of an alternation of flights and perchings." William James

To summarize, metastability in neuroscience refers to the ability of the brain to continuously switch between different functional networks in order to integrate information, even in resting state.

What is Metatability in statistical physics?

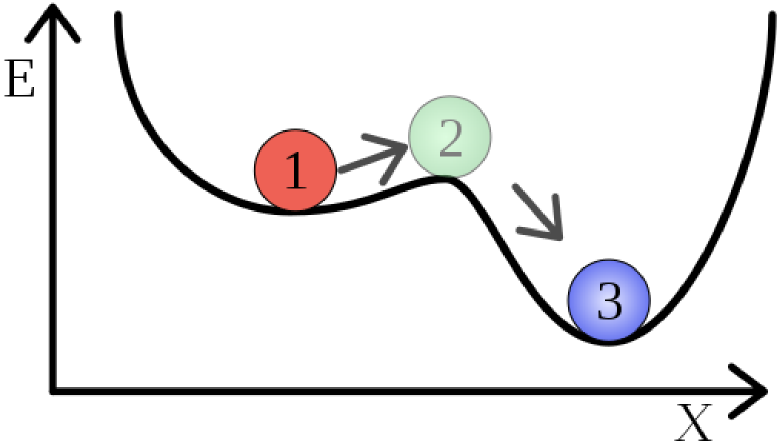


Figure 1: Metastable states can be seen as local minima of energy.

What is Metastability in statistical physics?

In a seminal paper O. Penrose and J. L. Lebowitz proposed the following characterization for a metastable thermodynamic state :

1. only one thermodynamic phase is present,
2. a system that starts in this state is likely to take a long time to get out,
3. once the system has gotten out, it is unlikely to return.

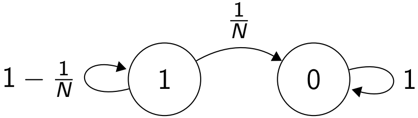
What is Metastability in statistical physics?

About ten years latter, M. Cassandro, A. Galves, E. Olivieri and M. E. Vares introduced in a refinement of the second point: the exit time shall be memory-less, that is, exponentially distributed.

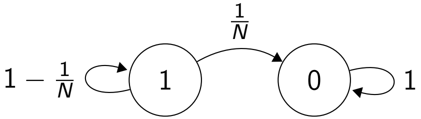
Examples: Curie-Weiss model, Contact process.

In this frame-work the "perturbation" doesn't come from an external environment, it is inherent to the system. It is an endogenous perturbation that comes from the occurrence of statistically rare event.

The simplest possible Metastable stochastic process



The simplest possible Metastable stochastic process



We denote by τ_N the number of steps (or the "time") it takes to reach 0 when starting in state 1.

The simplest possible Metastable stochastic process

The law of τ_N is the geometric distribution with probability of success $\frac{1}{N}$. That is

$$\mathbb{P}\left(\tau_N = k\right) = \left(1 - \frac{1}{N}\right)^{k-1} \frac{1}{N}.$$

The geometric law has the important property that it is the only discrete distribution which is memory-less. That is, the only one satisfying

$$\mathbb{P}\left(\tau_N > m + n \mid \tau_N > m\right) = \mathbb{P}\left(\tau_N > n\right).$$

The Exponential distribution

The systems we consider however are time continuous.

The equivalent of the geometric law in continuous time is the exponential law, defined by : $X \rightsquigarrow \mathcal{E}(1)$ if $\mathbb{P}(X \geq t) = e^{-t}$.

Like for the geometric law, it is the only continuous time distribution to be memory-less, that is, the only distribution for which

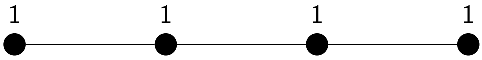
$$\mathbb{P}(X \geq t + s \mid X \geq t) = \mathbb{P}(X \geq s).$$

Metastability in continuous time

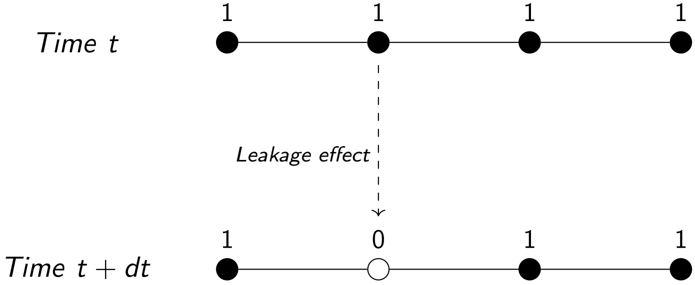
If τ_N now denotes the time of extinction of a continuous time system containing N individuals/neurons/particles, then the result of metastability we are looking for is the following.

$$\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow[N\to\infty]{\mathcal{D}} \mathcal{E}(1).$$

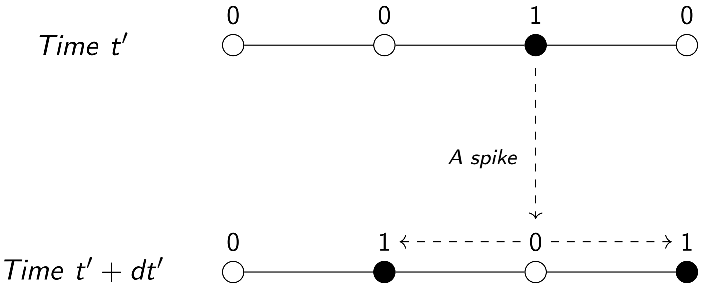
A system of spiking neurons, one dimensional lattice



A system of spiking neurons, one dimensional lattice



A system of spiking neurons, one dimensional lattice



Previous results on the one dimensional lattice

Let N be the number of neurons in the one dimensional lattice and γ the leakage rate.

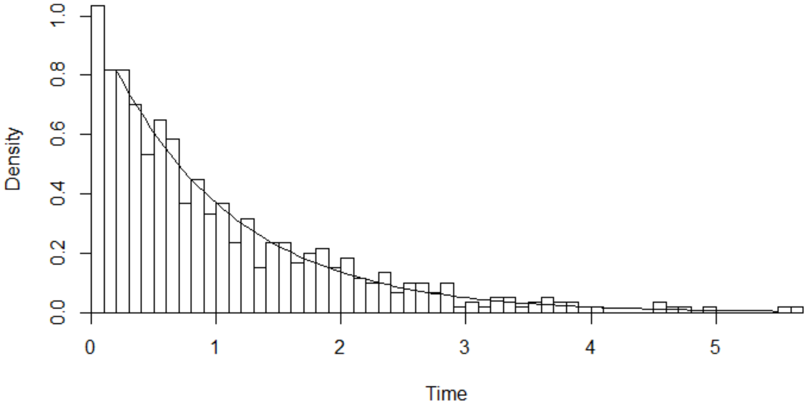
Theorem

There exists γ_c' such that if $0 < \gamma < \gamma_c'$, then we have the following convergence:

$$\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow[N\to\infty]{\mathcal{D}} \mathcal{E}(1).$$

Previous results on the one dimensional lattice

One dimensional lattice of size 100, $\gamma = 0.34$



Previous results on the one dimensional lattice

Let N be the number of neurons in the one dimensional lattice and γ the leakage rate.

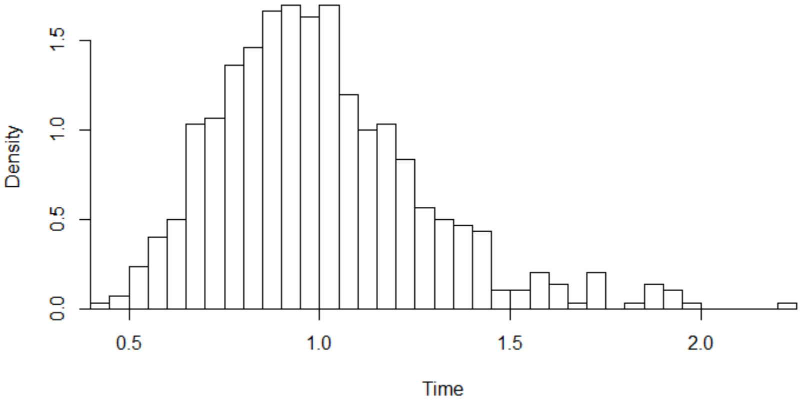
Theorem

Suppose that $\gamma > 1$. Then the following convergence holds

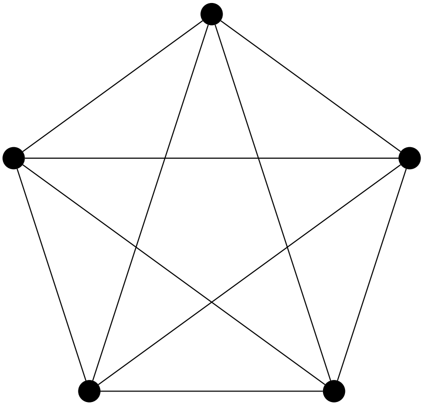
$$\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow[N\to\infty]{\mathbb{P}} 1.$$

Previous results on the one dimensional lattice

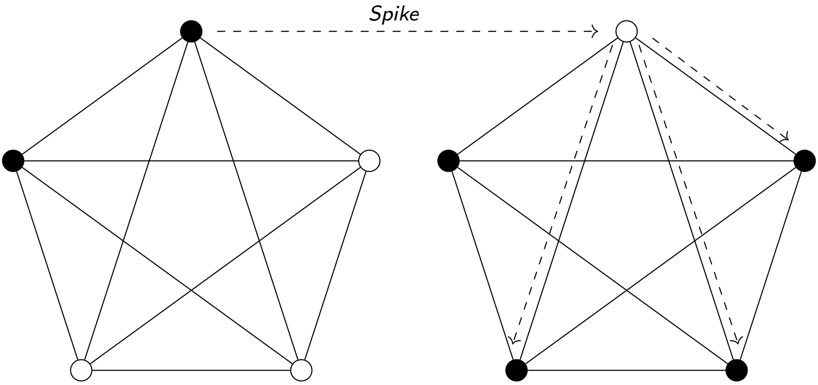
One dimensional lattice of size 100, $\gamma = 4$



A very peculiar network: the complete graph

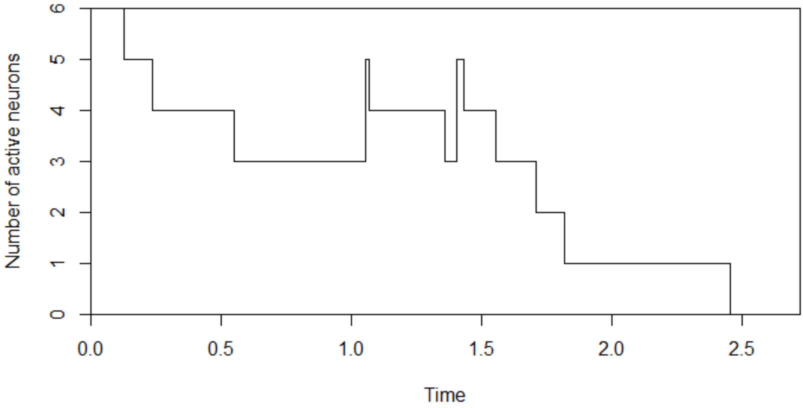


A very peculiar network: the complete graph



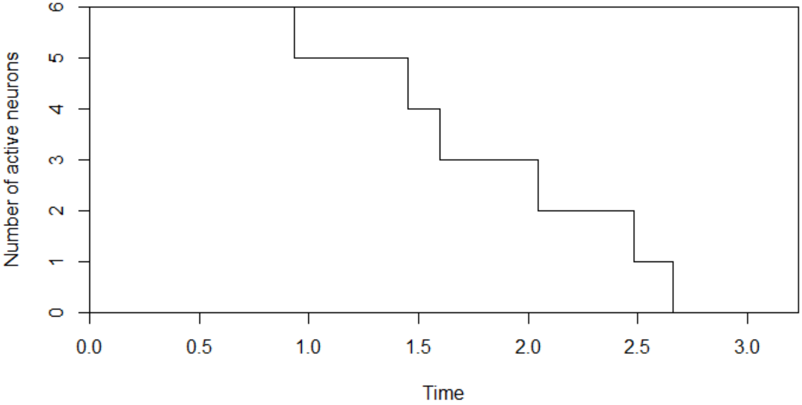
Complete graph and metastability

Complete graph of size 6, $\gamma = 4$



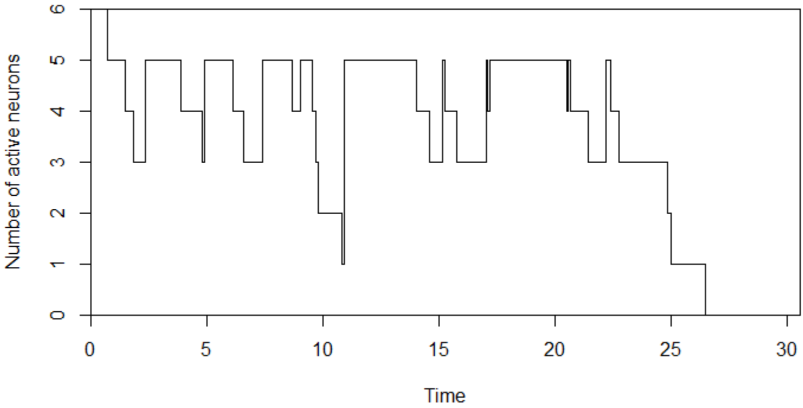
Complete graph and metastability

Complete graph of size 6, $\gamma = 2$



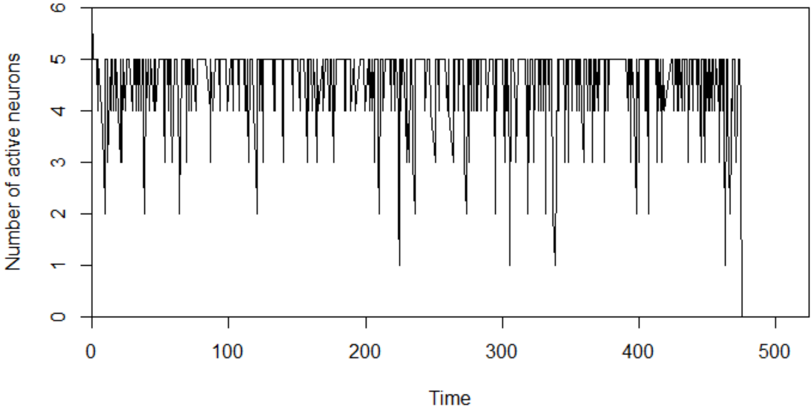
Complete graph and metastability

Complete graph of size 6, $\gamma = 1$



Complete graph and metastability

Complete graph of size 6, $\gamma = 0.5$



Results on the complete graph

Let N be the number of neurons in the complete graph and γ the leakage rate.

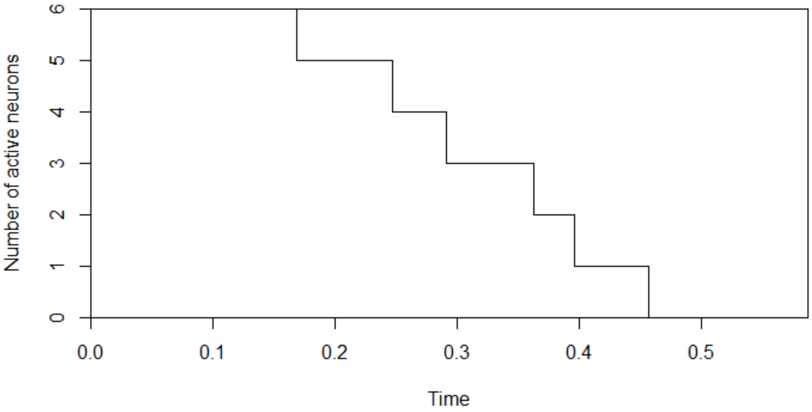
Theorem

For any γ strictly positive we have the following convergence:

$$\frac{\tau_N}{\mathbb{E}(\tau_N)} \xrightarrow[N\to\infty]{\mathcal{D}} \mathcal{E}(1).$$

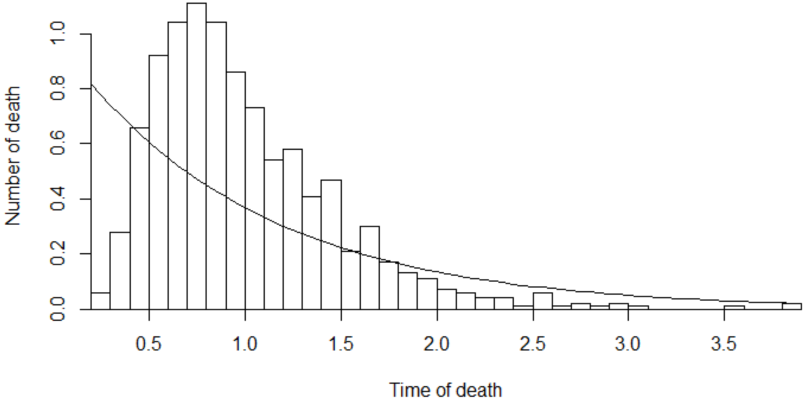
Complete graph and metastability

Complete graph of size 6, $\gamma = 8$



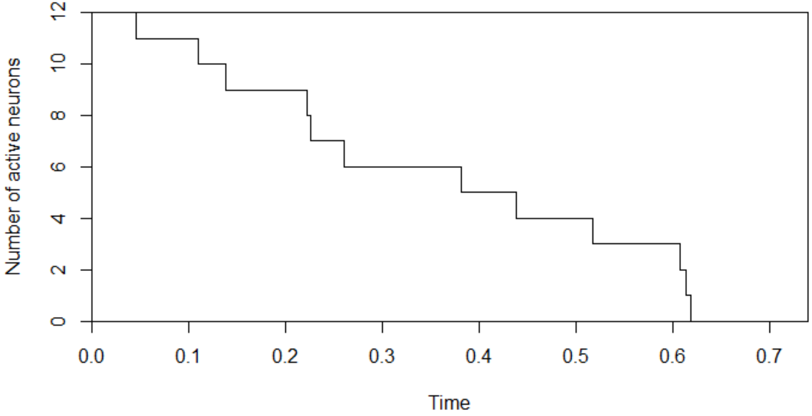
Complete graph and metastability

Complete graph of size 6, $\gamma = 8$



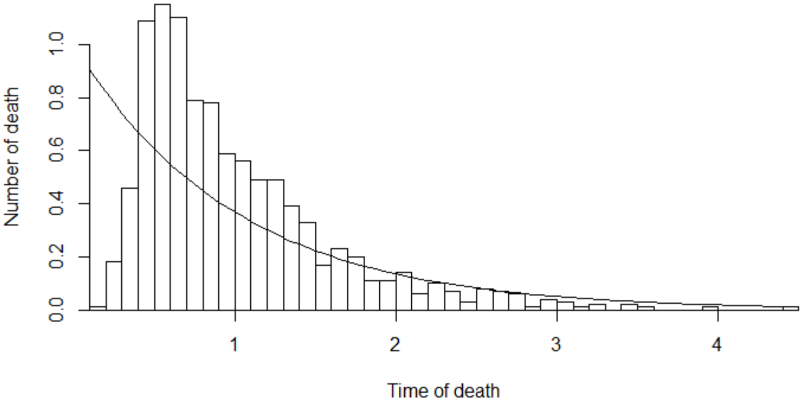
Complete graph and metastability

Complete graph of size 12, $\gamma = 8$



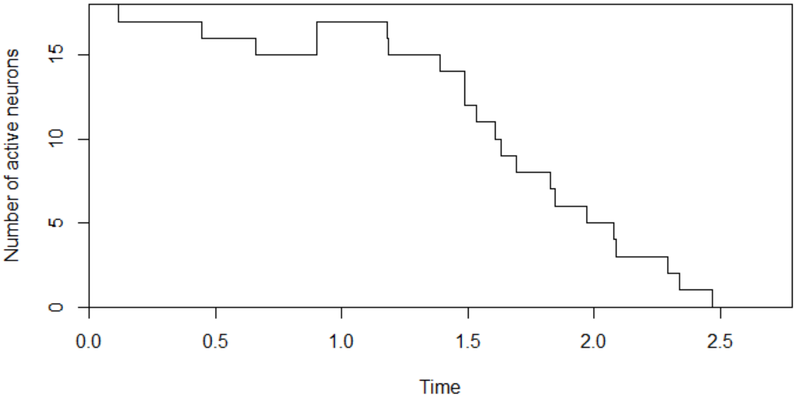
Complete graph and metastability

Complete graph of size 12, $\gamma = 8$



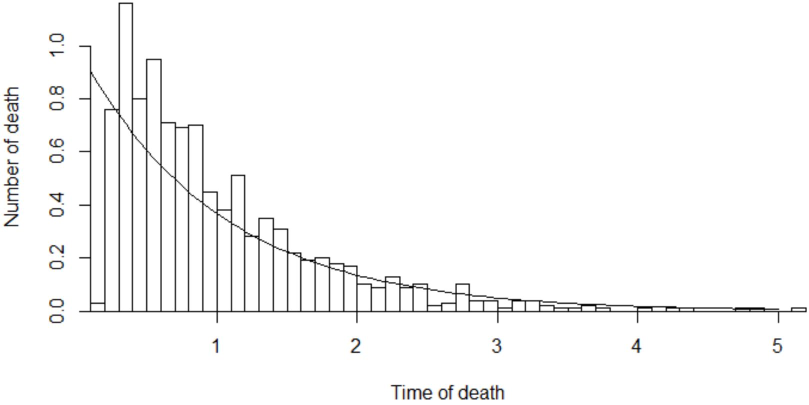
Complete graph and metastability

Complete graph of size 18, $\gamma = 8$



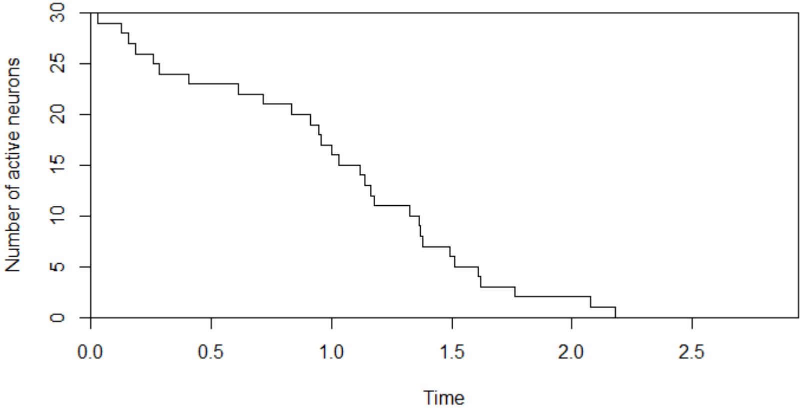
Complete graph and metastability

Complete graph of size 18, $\gamma = 8$



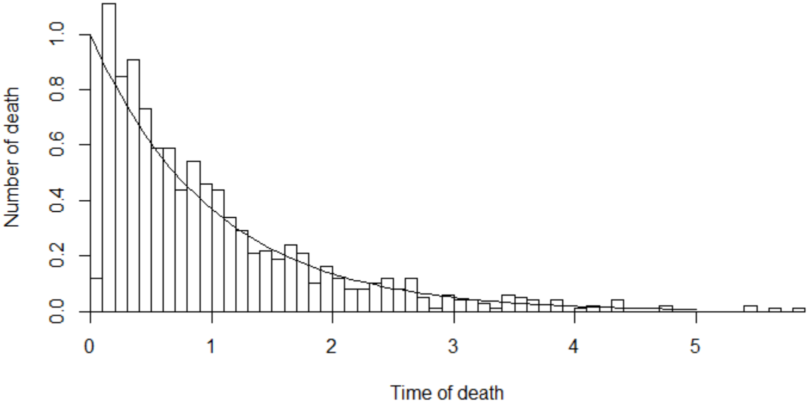
Complete graph and metastability

Complete graph of size 30, $\gamma = 8$



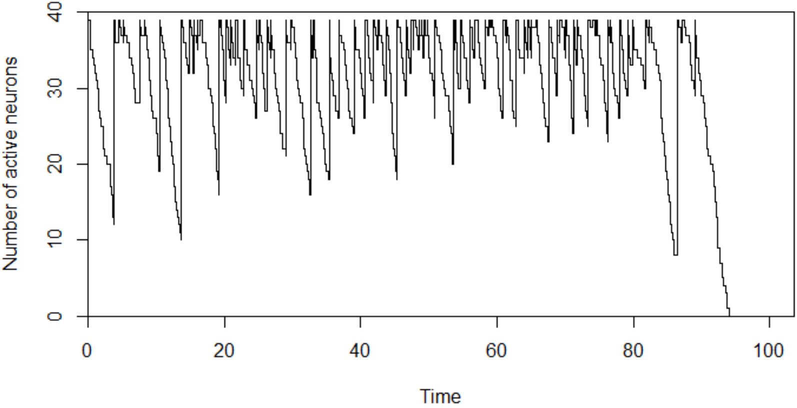
Complete graph and metastability

Complete graph of size 30, $\gamma = 8$



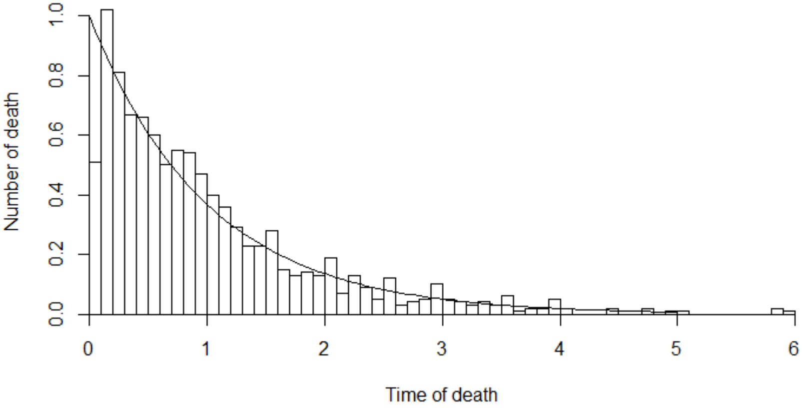
Complete graph and metastability

Complete graph of size 40, $\gamma = 8$



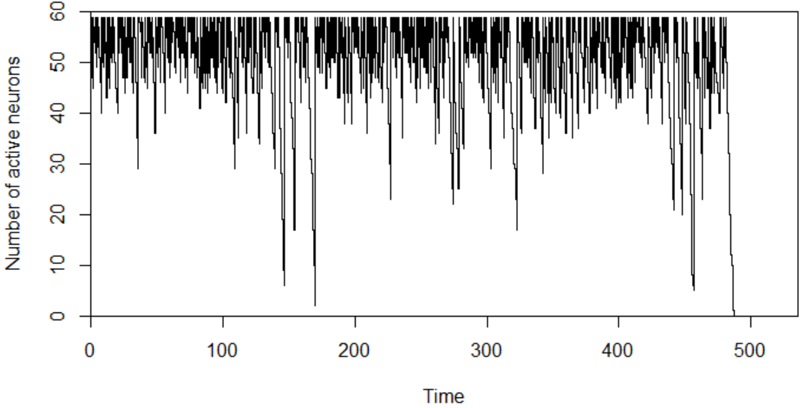
Complete graph and metastability

Complete graph of size 40, $\gamma = 8$



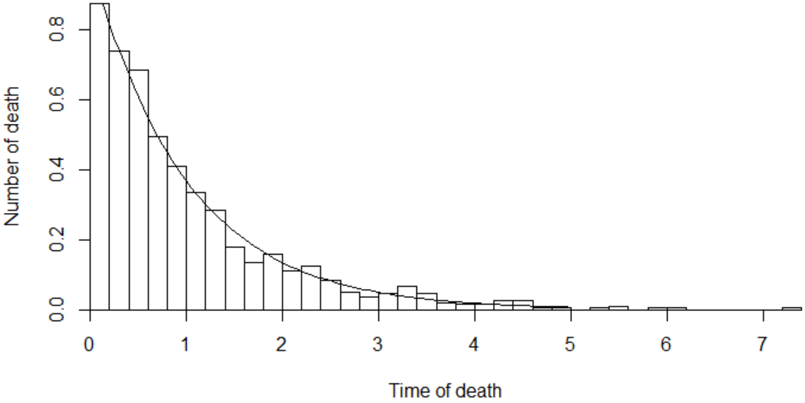
Complete graph and metastability

Complete graph of size 60, $\gamma = 8$



Complete graph and metastability

Complete graph of size 60, $\gamma = 8$



Thanks for your attention